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Quenching of hadron spectra in media

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Abstract: We determine how the yield of large transverse momentum hadrons is modified due to induced gluon radiation off a hard parton traversing a QCD medium. The quenching factor is formally a collinear- and infrared-safe quantity and can be treated perturbatively. In spite of that, in the $p_\perp$ region of practical interest, its value turns out to be extremely sensitive to large distances and can be used to unravel the properties of dense quark-gluon final states produced in heavy ion collisions. We also find that the quenching is not given by a simple shift of the hard parton cross section by the mean energy loss.

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Contents

1. Introduction

The so-called jet quenching [1]–[6] is considered an important signal of the production of a new state of dense matter (quark-gluon plasma) in ultrarelativistic heavy ion collisions. This is understood as the suppression of the yield of large transverse momentum jets or particles with respect to proton-proton collisions.

In this paper we concentrate on the quenching effect in inclusive particle spectra, due to the energy loss by medium induced gluon radiation [7]–[24].

Inclusive production of particles with large $p_\perp$, in, say, proton-proton collisions can be parametrized as a power

$$\frac{d\sigma^{\text{vacuum}}(p_\perp)}{dp_\perp^2} \propto \frac{1}{p_\perp^n}\,, \qquad n = n(p_\perp) \equiv -\frac{d}{d\ln p_\perp} \ln \frac{d\sigma^{\text{vacuum}}(p_\perp)}{dp_\perp^2} \tag{1.1}$$

with n an effective exponent which slowly decreases with increase of $p_\perp$. In reality, for moderately large $p_\perp$ the exponent in (1.1) is much larger than the asymptotic value $n = 4$ corresponding to the limit $p_\perp \to \infty$ ($s/p_\perp^2 = \text{const}$).

The effects that add to the value of n are:

1. x-dependence of the parton distributions which decrease with increase of parton energies, $x_1, x_2 \propto \sqrt{p_\perp^2/s}$,

2. the bias effect due to vetoing accompanying gluon radiation off the primary large-p_t parton that produces the triggered particle with $p_\perp \le p_t$,

3. running of the coupling $\alpha_s^2(p_\perp^2)$ in the parton-parton scattering cross section.

As a result, in practice n is seen to be as large as 10. In what follows we shall treat n as a large numerical parameter neglecting relative corrections of the order of $1/n$. (Such an approximation, though unnecessary, allows us to derive a simple analytic expression for jet quenching due to medium effects.)

In the presence of a medium the inclusive spectrum (1.1) changes. Multiple interactions of partons with the medium lead to two competing effects. On the one hand, multiple scattering in the initial (as well as in the final) state partially transforms longitudinal parton motion into transverse one, thus enhancing the yield of large-$p_\perp$ particles (the so-called Cronin effect [25]). On the other hand, medium induced gluon radiation accompanying multiple scattering causes parton energy loss and therefore suppresses the particle yield.

At large transverse momenta the second effect takes over.

The standard jet fragmentation involves gluon bremsstrahlung caused by the hard parton scattering, followed by the hadronization of the final parton (which we suppose to take place *outside* the medium). It reduces the energy of the originally produced parton p_t to that of the leading particle in the jet, $p_\perp < p_t$. These effects are already included in the vacuum cross section.[1] Since the additional medium induced energy loss turns out to be relatively small, $\epsilon \ll p_\perp$ (see below), we may treat it as an *additive* effect and thus avoid introducing the *in-medium* parton $\to$ hadron fragmentation function.

The correspondence between the parton and final hadron momenta in the vacuum $(p_t \to p_\perp)_{\rm vac}$ translates in the medium into $(p_t \to p_\perp - \epsilon)_{\rm med}$. Bearing this in mind, to find the inclusive particle spectrum with a given $p_\perp$ we have to convolute the vacuum production cross section of the particle with energy $p_\perp + \epsilon$ with the distribution $D(\epsilon)$ that describes specifically the *additional* energy loss ϵ due to medium induced gluon radiation in the final state:

$$\frac{d\sigma^{\rm medium}(p_\perp)}{dp_\perp^2} = \int d\epsilon\, D(\epsilon)\, \frac{d\sigma^{\rm vacuum}(p_\perp + \epsilon)}{dp_\perp^2} \,. \tag{1.2}$$

The quenching effect is customarily modelled by the substitution

$$\frac{d\sigma^{\rm medium}(p_\perp)}{dp_\perp^2} = \frac{d\sigma^{\rm vacuum}(p_\perp + S)}{dp_\perp^2} \,. \tag{1.3}$$

[1] For the sake of simplicity we consider particle production at $90^{\rm o}$ and equate the transverse momentum of the particle $p_\perp$ with its energy.

JHEP09(2001)033

The *shift* parameter S in (1.3) is usually taken either proportional to the size of the medium,

$$S = \text{const} \cdot L\,, \tag{1.4a}$$

or equal to the *mean* medium induced energy loss [11]

$$S = \Delta E \equiv \int d\epsilon\, \epsilon\, D(\epsilon) \propto \alpha_s\, L^2\,. \tag{1.4b}$$

The former ansatz has no theoretical justification while the latter emerges as a result of the Taylor expansion of (1.2) based on the $\epsilon \ll p_\perp$ approximation:

$$\begin{aligned} \int d\epsilon\, D(\epsilon) \cdot \frac{d\sigma(p_\perp + \epsilon)}{dp_\perp^2} &= \int d\epsilon\, D(\epsilon) \cdot \frac{d\sigma(p_\perp)}{dp_\perp^2} + \int d\epsilon\, \epsilon\, D(\epsilon) \cdot \frac{d}{dp_\perp} \frac{d\sigma(p_\perp)}{dp_\perp^2} + \cdots \\ &\simeq \frac{d\sigma}{dp_\perp^2} + \Delta E \cdot \frac{d}{dp_\perp}\left(\frac{d\sigma}{dp_\perp^2}\right) \simeq \frac{d\sigma(p_\perp + \Delta E)}{dp_\perp^2}\,. \end{aligned} \tag{1.5}$$

Such an approximation misses, however, one essential point, namely that the vacuum distribution is a sharply falling function of $p_\perp$. This causes a strong bias which leads to an additional suppression of real gluon radiation. As a result, the *typical* energy carried by accompanying gluons turns out to be much smaller than the *mean* (1.4b).

In this paper we study an interplay between the energy loss and the cross section fall-off and show that it leads, in the region of transverse momenta of practical interest, to the $p_\perp$ dependent expression for the shift

$$S(p_\perp) \propto \sqrt{p_\perp}\,.$$

2. Medium induced energy loss

2.1 Transport coefficient

The two effects — parton transverse momentum broadening and medium induced radiation are closely related and are determined by the so-called "transport coefficient" $\hat{q}$ which characterizes the "scattering power" of the medium [11]:

$$\hat{q}^{(R)} = \rho \int dq^2\, q^2\, \frac{d\sigma^{(R)}}{dq^2}\,. \tag{2.1}$$

Here ρ is the density of scattering centres, and $d\sigma^{(R)}$ is the single scattering cross section for a projectile parton in the colour representation R, with C_R the corresponding colour factor ($C_F = (N_c^2 - 1)/2N_c = 4/3$, $C_A = N_c = 3$ for quark and gluon, respectively).

The dimensionless ratio $\hat{q}/\rho$ characterizes the "opacity" of the medium for an energetic gluon. It includes the region of small momentum transfers where the perturbative treatment is hardly applicable, and should be regarded as (the only) unknown medium-dependent parameter of the problem.

In "cold" nuclear matter $\hat{q}$ can be calculated perturbatively and related to the gluon density $[xG(x,Q^2)]$ of the nucleus at a low momentum scale $Q^2 \simeq \hat{q}\,L$ and small but not too small x, where the gluon density has little dependence on x [11]:

$$\hat{q}^{(R)} \simeq \rho\, \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \left[xG(x, \hat{q}^{(R)} L) \right]. \tag{2.2}$$

Hereafter we shall label $\hat{q}$ the *gluon* transport coefficient. Taking $\rho = 0.16\ \mathrm{fm}^{-3}$, $\alpha_s = 0.5$ and $xG(x) = 1$ in (2.2) results in

$$\hat{q}_{\mathrm{cold}} \simeq 0.009\,\mathrm{GeV}^3 \simeq 0.045\,\frac{\mathrm{GeV}^2}{\mathrm{fm}}. \tag{2.3}$$

$\hat{q}$ enters, in particular, as the proportionality factor between an accumulated parton transverse momentum squared and the size of the medium traversed, $\kappa^2 \propto \hat{q}L$. The quark transport coefficients extracted from experimentally measured transverse momentum nuclear broadening of Drell-Yan lepton pairs [26] agrees with the theoretical estimate (2.3).

In the case of heavy ion collisions, the scattered hard parton traverses a medium that is expected to have an energy density much higher than that of nuclear matter, and the corresponding transport coefficient $\hat{q}_{\mathrm{hot}}$ can be much larger. If hot matter is formed in the final state, a perturbative estimate for the QGP with $T = 250\,\mathrm{MeV}$ gives [11]

$$\hat{q}_{\mathrm{hot}} \simeq 0.2\,\mathrm{GeV}^3 \simeq 1\,\frac{\mathrm{GeV}^2}{\mathrm{fm}}. \tag{2.4}$$

2.2 Induced gluon radiation

To find the distribution $D(\epsilon)$ in the parton energy loss ϵ in the final state we need to recall the basic properties of gluon radiation caused by multiple parton scattering in the medium.

Introducing the characteristic gluon frequency

$$\omega_c = \frac{\hat{q}}{2}\,L^2, \tag{2.5}$$

with L the length of the medium, the *inclusive* energy spectrum of medium induced soft gluon radiation ($\omega \ll p_\perp$) reads [12, 13]

$$\frac{dI(\omega)}{d\omega} = \frac{\alpha}{\omega} \ln \left| \cos \sqrt{\frac{i\,\omega_c}{\omega}} \right| = \frac{\alpha}{2\omega} \ln \left[\cosh^2 \sqrt{\frac{\omega_c}{2\omega}} - \sin^2 \sqrt{\frac{\omega_c}{2\omega}} \right]; \quad \alpha \equiv \frac{2\alpha_s C_R}{\pi}. \tag{2.6}$$

This distribution peaks at small gluon energies,

$$\omega \frac{dI(\omega)}{d\omega} = \alpha \left\{ \sqrt{\frac{\omega_c}{2\omega}} - \ln 2 \right\} \cdot \left[1 + \mathcal{O}\left(\exp\left\{ -\sqrt{\frac{2\omega_c}{\omega}} \right\} \right) \right], \qquad \omega < \omega_c, \tag{2.7}$$

while for energies above the characteristic scale it is small and falling fast with ω:

$$\omega \frac{dI(\omega)}{d\omega} \simeq \frac{\alpha}{12} \left(\frac{\omega_c}{\omega}\right)^2 , \qquad \omega > \omega_c \,. \tag{2.8}$$

The multiplicity of gluons with energies larger than a given ω is given by the integal of the inclusive gluon spectrum (2.6):

$$N(\omega) \equiv \int_\omega^\infty d\omega' \, \frac{dI(\omega')}{d\omega'} = \alpha \int_0^{\sqrt{\omega_c/2\omega}} \frac{dz}{z} \ln\left(\cosh^2 z - \sin^2 z\right) . \tag{2.9}$$

Evaluating this integral for $x = \omega/\omega_c \ll 1$ we have (cf. (2.7))

$$N(\omega) \simeq \alpha \left[\sqrt{\frac{2}{x}} + \ln 2 \ln x - 1.44136 + \mathcal{O}\left(\exp(-\sqrt{2/x})\right) \right] , \tag{2.10}$$

with the constant term found by numerical integration of the exact spectrum (2.6).

2.3 Energy loss distribution

Since the vacuum spectrum (1.1) is falling fast with $p_\perp$, the bias effect forces the distribution $D(\epsilon)$ to the smallest energy losses possible, $\epsilon/p_\perp \ll 1$. In these circumstances the "final" parton is the one that had been produced in the hard interaction: the quark-gluon transition is additionally suppressed as $\epsilon/p_\perp$ and can be neglected. Bearing this in mind, we will treat $D(\epsilon)$ as the distribution in energy that the hard parton (a quark or a gluon) loses to medium induced gluon bremsstrahlung.

The spectrum of the leading particle can be characterised by the probability $D(\epsilon)$ that the radiated gluons carry altogether a given energy ϵ. The corresponding expression based on independent emission of soft primary gluons reads

$$D(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int d\omega_i \, \frac{dI(\omega_i)}{d\omega} \right] \delta\left(\epsilon - \sum_{i=1}^{n} \omega_i \right) \cdot \exp\left[- \int d\omega \frac{dI}{d\omega} \right] , \tag{2.11}$$

where the last factor accounts for virtual effects. In a standard way, the energy constraint can be factorized using the Mellin representation,

$$\delta\left(\epsilon - \sum_{i=1}^{n} \omega_i \right) = \int_C \frac{d\nu}{2\pi i} e^{\nu\epsilon} \cdot \prod_{i=1}^{n} e^{-\nu\omega_i} , \tag{2.12}$$

after which multiple gluon radiation exponentiates, and the answer can be written as

$$D(\epsilon) = \int_C \frac{d\nu}{2\pi i} \tilde{D}(\nu) \, e^{\nu\epsilon} , \tag{2.13a}$$

$$\tilde{D}(\nu) = \exp\left[- \int_0^\infty d\omega \, \frac{dI(\omega)}{d\omega} \left(1 - e^{-\nu\omega}\right) \right] . \tag{2.13b}$$

JHEP09(2001)033

The contour C in (2.12) and (2.13a) runs parallel to the imaginary axis in the complex ν-plane, $\mathrm{Re}\,\nu$ = const. An equivalent expression can be written in terms of the integrated gluon multiplicity (2.9). Integrating (2.13b) by parts, we obtain an elegant formula

$$\tilde{D}(\nu) = \exp\left[-\nu\int_0^\infty d\omega\, e^{-\nu\omega}\, N(\omega)\right] \tag{2.14a}$$

$$= \exp\left[-\int_0^\infty dz\, e^{-z}\, N\left(\frac{z}{\nu}\right)\right]. \tag{2.14b}$$

As we shall see shortly, the value of the dimensional variable $\nu\omega_c$ is linked with the characteristic parameter of the problem $n\omega_c/p_\perp$ which is typically much larger than unity. Therefore, in the essential integration region in (2.13b) we have $\omega \sim 1/\nu \sim p_\perp/n \ll \omega_c$, and the approximate expression (2.10) can be used. This gives

$$\tilde{D}(\nu) \simeq \exp\left\{-\alpha\left(\sqrt{2\pi\nu\omega_c} - \ln 2\ln(\nu\omega_c) - 1.84146\right)\right\}, \tag{2.15}$$

which expression allows one to evaluate the energy loss distribution $D(\epsilon)$ for $\epsilon \ll \omega_c$ analytically in terms of the hypergeometric function.

For the purpose of illustration we present here only a rough estimate based on the leading small-energy behaviour of the gluon distribution $\omega dI/d\omega \propto \alpha_s\sqrt{\omega_c/\omega}$ which translates into

$$\tilde{D}(\nu) \simeq \exp\left\{-\alpha\sqrt{2\pi\nu\omega_c}\right\}. \tag{2.16}$$

The Laplace integral (2.13a) then becomes gaussian and yields

$$\epsilon D(\epsilon) \simeq \alpha\sqrt{\frac{\omega_c}{2\epsilon}}\exp\left\{-\frac{\pi\alpha^2\omega_c}{2\epsilon}\right\}. \tag{2.17}$$

This approximation is fine for illustrative purposes, in particular because the distribution (2.17) is properly normalized to unity due to $\tilde{D}(\nu = 0) = 1$ in (2.16). As expected, in the first order in α_s the energy loss spectrum coincides with the probability of emission of one gluon with the energy $\omega = \epsilon$, see (2.7). The one-gluon approximation spectacularly fails, however, at small ϵ. If the energy loss is taken as small as $\epsilon \sim \alpha^2\omega_c$, the distribution reaches its maximum at

$$\epsilon = \pi\alpha^2\,\omega_c \ll \omega_c \tag{2.18}$$

and becomes exponentially small at smaller energies, due to form factor suppression.

3. Quenching

We introduce the medium dependent *quenching factor* Q to represent the inclusive particle spectrum as

$$\frac{d\sigma^{\mathrm{medium}}(p_\perp)}{dp_\perp^2} = \frac{d\sigma^{\mathrm{vacuum}}(p_\perp)}{dp_\perp^2}\cdot Q(p_\perp) \tag{3.1}$$

with

$$Q(p_\perp) = \int d\epsilon\, D(\epsilon) \cdot \left(\frac{d\sigma^{\text{vacuum}}(p_\perp + \epsilon)/dp_\perp^2}{d\sigma^{\text{vacuum}}(p_\perp)/dp_\perp^2} \right). \tag{3.2}$$

Using the fact that the effective exponent n in the ratio

$$R = \frac{d\sigma^{\text{vacuum}}(p_\perp + \epsilon)/dp_\perp^2}{d\sigma^{\text{vacuum}}(p_\perp)/dp_\perp^2} \simeq \left(\frac{p_\perp}{p_\perp + \epsilon} \right)^n$$

entering the convolution (3.2) is numerically large, we can replace R by the exponential form

$$R = \exp\left(-\frac{n\epsilon}{p_\perp} \right) \cdot \left[1 + \mathcal{O}\left(\frac{\ln^2 R}{2\,n} \right) \right]. \tag{3.3}$$

The accuracy of such a substitution is rather good all over the region of practical interest where the quenching suppression is moderately large.

The exponential approximation (3.3) results in a simple expression for the quenching, as the suppression factor equals a given Mellin moment in ϵ of the spectrum $D(\epsilon)$ (see (2.13a) and (2.14)):

$$\begin{aligned} Q(p_\perp) &\simeq \int_0^\infty d\epsilon\, D(\epsilon)\, \exp\left\{ -\frac{n}{p_\perp} \cdot \epsilon \right\} \\ &= \tilde{D}\left(\frac{n}{p_\perp} \right) = \exp\left\{ -\int_0^\infty dz\, e^{-z} N\left(\frac{p_\perp}{n} z \right) \right\}. \end{aligned} \tag{3.4}$$

Representing the quenching factor as

$$Q(p_\perp) = \exp\left\{ -\frac{n}{p_\perp} \cdot S(p_\perp) \right\}, \tag{3.5a}$$

$$S(p_\perp) \equiv -\frac{p_\perp}{n} \ln \tilde{D}\left(\frac{n}{p_\perp} \right) = \int_0^\infty d\omega\, N(\omega)\, \exp\left\{ -\frac{n\,\omega}{p_\perp} \right\}, \tag{3.5b}$$

and using again the large-n approximation, we can cast the quenching effect as a *shift* of the vacuum spectrum:

$$\frac{d\sigma^{\text{medium}}(p_\perp)}{dp_\perp^2} \simeq \frac{d\sigma^{\text{vacuum}}(p_\perp + S(p_\perp))}{dp_\perp^2}. \tag{3.6}$$

It should be noted that the shift approximation (3.6) *underestimates* suppression if the quenching factor Q becomes very small.

The general expression (3.4) actually solves the quenching problem, given the gluon emission spectrum $dI/d\omega$. To verify the validity of the approach and the approximations made, we need, however, to estimate the characteristic gluon energies that determine the answer. To this end we proceed with a qualitative analysis of QCD quenching.

Using the small-energy approximation (2.10) in (3.4) we derive

$$Q(p_\perp) \simeq \exp\left\{ -\sqrt{\pi}\, N\left(\frac{p_\perp}{n} \right) \right\} \simeq \exp\left\{ -N\left(\frac{p_\perp}{\pi\, n} \right) \right\}. \tag{3.7}$$

JHEP09(2001)033

This expression has a simple physical interpretation. It is a typical exponential form factor suppression whose exponent equals the probability of gluon radiation in the *forbidden* kinematical region or, in other words, the mean multiplicity of virtual gluons. We see that the energies of *real* gluons are cut from above as

$$\omega < \frac{\omega_1}{\pi}\,, \qquad \omega_1 = \frac{p_\perp}{n}\,, \tag{3.8}$$

which (modulo a constant[2] factor π) equals a small ($1/n$) fraction of the energy of the registered particle, due to the bias effect. The characteristic energy parameter of the quenching problem can be directly expressed in terms of the hard vacuum cross section by the general relation

$$\omega_1(p_\perp) \equiv -\left[\frac{d}{dp_\perp}\,\ln\frac{d\sigma^{\rm vacuum}}{dp_\perp^2}\right]^{-1}\,, \tag{3.9}$$

which does not rely on the power approximation (1.1) with n =const.

For the "shift" function (3.5) in the approximation (2.10) we obtain

$$S(p_\perp) \simeq \sqrt{\frac{2\pi\,\alpha^2\,\omega_c p_\perp}{n}}\,. \tag{3.10}$$

This result namely, the $p_\perp$ dependent shift, is very different from the models (1.4) that were discussed in the literature and are being used to predict/describe quenching [1]–[6]. To understand the origin of (3.10) in the rest of this section we consider and compare the mean energy loss with a typical energy loss that characterizes quenching.

Mean energy loss. The *mean* medium induced energy loss of a parton with a given energy E is determined by the integral

$$\Delta E \equiv \int_0^\infty d\epsilon\,\epsilon\,D(\epsilon) = -\left.\frac{d}{d\nu}\tilde{D}(\nu)\right|_{\nu=0} = \int_0^\infty d\omega\,\omega\,\frac{dI(\omega)}{d\omega} = \int_0^\infty d\omega\,N(\omega)\,, \tag{3.11}$$

where we have used (2.13). Given $\omega dI/d\omega \propto N(\omega) \propto \omega^{-1/2}$, this integral is determined by the largest available gluon energies, $\omega \lesssim \omega_{\rm max}$, resulting in [11]

$$\Delta E \propto \alpha_s\sqrt{\omega_c\,\omega_{\rm max}}\,, \qquad \omega_{\rm max} = \min\{\omega_c\,,\,E\}\,. \tag{3.12}$$

The commonly used identification of the shift with the mean energy loss (1.4b) is valid in the region of large particle energies $p_\perp \gtrsim n\omega_c$, corresponding to the quenching parameter $\nu\omega_c \lesssim 1$. Indeed, since the integrated gluon multiplicity $N(\omega)$ vanishes fast for $\omega > \omega_c$, for $\nu < 1/\omega_c$ we can omit the exponential factor in the integrand of (2.14a) to derive

$$S(p_\perp) \simeq \int_0^\infty d\omega\,N(\omega) \equiv \Delta E\,. \tag{3.13}$$

[2]dimensional, some would add,

In this region, however, the quenching itself is vanishingly weak since $\Delta E \propto \alpha_s \omega_c$ and

$$-\ln Q(p_\perp) = \frac{n}{p_\perp} \cdot \Delta E \propto \alpha_s \cdot \frac{n\omega_c}{p_\perp} < \alpha_s \,.$$

Gluons responsible for the mean energy loss are rare, $\mathcal{O}(\alpha_s)$. Strictly speaking, rare fluctuations with energetic gluons ($\omega \gtrsim \omega_c$) in $dI/d\omega$ do contribute to quenching via the virtual suppression factor in (2.13a):

$$\delta \ln Q = -\int_{\omega_c}^{\infty} d\omega \, \frac{dI(\omega)}{d\omega} \,.$$

This contribution, however, is bound to be small (negligible) as long as the mean *multiplicity* (not mean *energy*!) of such gluons is of the order α_s.

This argument applies both to the "canonical" ΔE [12, 13] and to the additional contribution to ΔE originating from emission of energetic gluons when there is only a *single* scattering in the medium, found in a series of recent papers initiated by Gyulassy, Lévai and Vitev [20]–[23]. The GLV energy spectrum has an enhanced high-energy tail,

$$\frac{\omega\, dI^{(\mathrm{GLV})}(\omega)}{d\omega} \propto \alpha \, \frac{\omega_c}{\omega} \,, \quad \omega > \omega_c \qquad \left[\text{cf.} \quad \frac{\omega\, dI^{(\mathrm{BDMPS})}(\omega)}{d\omega} \propto \alpha \left(\frac{\omega_c}{\omega}\right)^2 \right],$$

thus inducing a potentially large contribution to ΔE from the region $\omega \gg \omega_c$. Such fluctuations, however, do not affect quenching since

$$\int_{\omega_c}^{\infty} d\omega \, \frac{dI^{(\mathrm{GLV})}(\omega)}{d\omega} = \mathcal{O}(\alpha_s) \,.$$

Typical energy loss. To have a significant quenching, $-\ln Q(p_\perp) = \mathcal{O}(1)$, we have to have $N(p_\perp/n) \sim 1$, according to (3.4), which translates into $p_\perp \lesssim \alpha^2 n\omega_c \ll n\omega_c$.

The value of the convolution integral (1.2) that determines quenching results in an interplay of the steep fall-off of the parton cross section with ϵ and the form factor suppression of small losses. To estimate the characteristic energy loss for a given $p_\perp$ we invoke the approximate expression (2.17) for the distribution $D(\epsilon)$ to write

$$Q(p_\perp) \simeq \frac{\alpha}{\sqrt{2}} \int_0^{\infty} \frac{dx}{x^{\frac{3}{2}}} \exp\left\{ -\frac{\pi\alpha^2}{2\,x} - \frac{n\omega_c}{p_\perp} x \right\} = \exp\left\{ -2\sqrt{\frac{\pi\alpha^2 n\omega_c}{2\,p_\perp}} \right\}, \quad x \equiv \frac{\epsilon}{\omega_c} \,. \tag{3.14a}$$

In the region where the quenching is strong, $Q \ll 1$, the steepest descent evaluation applies, provided the exponent on the r.h.s. of (3.14a) is large.

With $p_\perp$ increasing, quenching becomes weak, $1 - Q(p_\perp) \ll 1$. In this kinematical region we use the fact that $D(\epsilon)$ is normalized to unity to write an equivalent

JHEP09(2001)033

representation

$$1 - Q(p_\perp) \simeq \sqrt{\frac{\alpha^2 n\omega_c}{2\,p_\perp}} \int_0^\infty \frac{dy}{y}\,\frac{1-e^{-y}}{\sqrt{y}} \cdot \exp\left\{-\frac{\pi\alpha^2 n\omega_c}{2p_\perp\, y}\right\}, \qquad y = \frac{n\,\epsilon}{p_\perp}. \tag{3.14b}$$

Now, if the exponent in the last factor is small it can be dropped, and the integral over y is determined by $y = \mathcal{O}\,(1)$.

Combining the two estimates we arrive at

$$\langle\epsilon\rangle \simeq \sqrt{\frac{\pi\alpha^2\omega_c\, p_\perp}{2\,n}} \qquad \text{for} \qquad p_\perp < \frac{\pi}{2}\,\alpha^2\, n\,\omega_c\,, \tag{3.15a}$$

$$\langle\epsilon\rangle \simeq \frac{p_\perp}{n} \qquad \text{for} \qquad p_\perp > \frac{\pi}{2}\,\alpha^2\, n\,\omega_c\,, \tag{3.15b}$$

where $\langle\epsilon\rangle$ is the typical energy value dominating the integrals (3.14). The two expressions (3.15) match at the border value.

Typical energy loss and the shift. In the first regime (3.15a) gluon radiation is omnipresent. Invoking (3.5) we observe that the characteristic energy loss $\langle\epsilon\rangle$ in (3.15a) equals *half* of the shift function $S(p_\perp)$. The factor $\frac{1}{2}$ may look anti-intuitive at a first glance. To appreciate it we need to recall that the substitution

$$\frac{d\sigma^{\mathrm{medium}}(p_\perp)}{dp_\perp^2} \Longrightarrow \frac{d\sigma^{\mathrm{vacuum}}(p_\perp + \langle\epsilon\rangle)}{dp_\perp^2}$$

accounts only for a part of quenching: the second ingredient of the convolution (1.2) — the energy loss distribution $D\left(\langle\epsilon\rangle\right)$ — also supplies an (equal) exponential suppression factor, giving

$$S(p_\perp) = 2\,\langle\epsilon\rangle = 2\cdot\sqrt{\frac{\pi\alpha^2\omega_c\, p_\perp}{2\,n}}\,.$$

In the complementary region of larger $p_\perp$, the situation changes. Here gluon radiation is rare and can be treated as a correction. In the one-gluon approximation the loss is equal the gluon energy, $\langle\epsilon\rangle = \omega_1$ in (3.15b). It has to be multiplied by the (small) gluon probability (multiplicity) $\Delta M(\omega_1)$,

$$\Delta M(\omega) = \mathrm{const}\cdot\left[\omega\frac{dI(\omega)}{d\omega}\right] \tag{3.16a}$$

to obtain the shift (3.10),

$$S(p_\perp) = \omega_1\cdot\Delta M(\omega_1)\,. \tag{3.16b}$$

Characteristic single gluon energy. In (3.8) we have introduced the characteristic (single gluon) energy ω_1 as a border separating real and virtual emissions. Real and virtual gluons *softer* than ω_1 cancelled, while the multiplicity of *harder* virtual gluons provided the form factor suppression which resulted in quenching according to (3.4). Given an interpretation of ω_1 as the maximal energy of *real* gluons, the shift can again be expressed as in (3.16b) which evaluation is now valid for arbitrary $p_\perp$, across (3.15).

4. Approximations

We start the discussion of the approximations made by noting that having written the factorised convolution (1.2) we supposed that the leading quark turns into the registered hadron (hadronizes) *outside* the medium, which implies

$$p_\perp R_{\rm conf}^2 > L\,.$$

Then, the power approximation for the vacuum spectrum with n =const can be justified. Our derivation could have been damaged if the effective exponent n in (1.1) changed significantly over the energy range $p_\perp \div p_\perp + \epsilon$ in the essential region in ϵ. Substituting the typical energy loss (3.15a) we observe that it provides a correction

$$p_\perp + \langle\epsilon\rangle = p_\perp \left(1 + \frac{\ln Q^{-1}}{2\,n}\right)$$

which is actually *relatively small* for all values of Q of practical interest. This estimate allows us to evaluate n at the value of the transverse momentum of the registered particle, $n = n(p_\perp)$, and use the general expression (3.9) as the definition of the characteristic energy parameter ω_1.

Soft approximation. The medium induced gluon distribution (2.6) is valid in the soft gluon approximation, $\omega \ll p_\perp$. This approximation is justified by the fact that the main contribution to (3.7) comes from gluons with energies

$$\omega \sim \omega_1 = \frac{p_\perp}{n} \ll p_\perp\,. \tag{4.1}$$

Moreover, finite quenching, $\ln Q^{-1} = \mathcal{O}\,(1)$, implies $p_\perp \lesssim \alpha^2\, n\omega_c$ so that

$$\omega \sim \omega_1 = \frac{p_\perp}{n} \lesssim \alpha^2\,\omega_c \ll \omega_c\,, \tag{4.2}$$

so that the approximate expressions (2.7), (2.10) are legitimate.[3]

Independent gluon radiation. The master equation (2.11) was based on the independent gluon radiation picture. To estimate significance of possible interference effects we need to look at a typical density of gluons in configuration space.

To this end we recall that the lifetime (formation time) t of a medium induced gluon with a given energy ω and transverse momentum $k_\perp$ follows from the relations

$$t \sim \frac{\omega}{k_\perp^2}\,, \qquad k_\perp^2 \sim \frac{t}{\lambda}\,\mu^2\,, \tag{4.3}$$

[3]Subleading corrections due to hard gluons will be discussed in the next section.

JHEP09(2001)033

where μ is a typical transverse momentum transfer in a single scattering and λ the gluon mean free path. This gives ($\mu^2/\lambda \simeq \hat{q}$)

$$t \simeq \sqrt{\frac{\omega}{\hat{q}}}\,, \tag{4.4a}$$

$$k_\perp^2 \simeq \sqrt{\omega\,\hat{q}}\,. \tag{4.4b}$$

Multiplying (4.4a) by the multiplicity density (3.16a) and dividing by the size of the medium, we estimate the "gluon occupation number",

$$\text{Occup} \sim t \cdot \Delta M \cdot \frac{1}{L} \sim \sqrt{\frac{\omega}{\hat{q}}} \cdot \alpha \sqrt{\frac{2\omega_c}{\omega}} \cdot \frac{1}{L} = \alpha\,.$$

This looks natural: a bare quark produced in a hard interaction builds up its wave function by emitting gluons, with α being the gluon density per unit phase space. The rôle of a (dense) medium is to strip the quark off these gluons, so that the formation process repeats again and again, resulting in induced production of many gluons.

The smallness of the overlap between gluons[4] allows us to neglect possible interference effects in multiple gluon radiation as potentially contributing at the level of $\mathcal{O}(\alpha_s)$, while the main effect of the resummed medium induced radiation in (2.11) is $\mathcal{O}(1)$. This justifies, aposteriori, the Poisson approximation (2.11).

Scale of the coupling. According to (4.4b) the characteristic scale of the QCD coupling $\alpha_s(k_\perp)$ in the gluon emission spectrum (2.6) is rather small. For typical gluon energies (3.8) we have ($\hat{q} = 0.2\,\mathrm{GeV}^3$)

$$k_{1\perp} \sim \left(\frac{\hat{q}}{n}\, p_\perp\right)^{1/4} \simeq 700\,\mathrm{MeV} \cdot \left(\frac{p_\perp}{n \cdot \mathrm{GeV}},\right)^{1/4} = \mathcal{O}(1\,\mathrm{GeV})\,,$$

which value is rather low and marginally increases with $p_\perp$ reaching $k_{1\perp} \sim 1.5\,\mathrm{GeV}$ for $p_\perp \sim 100\,\mathrm{GeV}$ ($n \simeq 6$). In what follows we take a fixed value for the coupling

$$\alpha_s = 0.5\,, \quad \alpha = \frac{2\,\alpha_s\, C_F}{\pi} \simeq 0.42\,,$$

motivated by the studies of the effective QCD interaction strength in the infrared region.

Bethe-Heitler limit. The perturbative approach to the problem inevitably limits gluon energies from below. In particular, the gluon spectrum (2.6) describes the Landau-Pomeranchuk-Migdal (LPM) suppression of the independent Bethe–Heitler

[4] A similar argument applies to an interference between *medium*- and *vacuum*-produced gluons.

radiation and only applies to gluons with lifetimes larger than the mean free path, $t > \lambda$, which is equivalent to limiting gluon energies from above, see (4.4a),

$$\omega > \omega_{\rm BH} \sim \mu^2\lambda \sim \hat{q}\,\lambda^2\,. \tag{4.5}$$

In spite of the fact that the perturbative answer for the quenching factor (3.4) is formally infrared safe, its sensitivity to the region of small gluon energies may be significant, since the characteristic energy scale of the problem is strongly reduced by the bias effect. Indeed, according to (3.7), to keep the value of the quenching factor fully under perturbative control we have to choose the transverse momentum well above

$$p_\perp > \pi\, n\, \omega_{\rm BH} \approx 10\,{\rm GeV}\,, \tag{4.6}$$

where for the sake of estimate we have taken the vacuum exponent [3, 27] $n \simeq 12 \div 13$ and $\omega_{\rm BH} \simeq 300\,{\rm MeV}$ corresponding to a hot medium with $\hat{q} \simeq 0.2\,{\rm GeV}^3$ and $\lambda \simeq \frac{1}{4}$ fm. At gluon energies comparable to $\omega_{\rm BH}$ Debye screening effect should also become important thus limiting our ability of doing reliable calculations.

To estimate the absolute minimal value of the quenching factor we take $p_\perp \lesssim n\,\omega_{\rm BH}$ and, keeping in mind that $N(\omega)$ flattens out at small energies, from (3.4) obtain

$$\ln\frac{1}{[\,Q(p_\perp)\,]_{\min}} \simeq N(\omega_{\rm BH}) \simeq \frac{2\alpha_s\,C_R}{\pi}\sqrt{\frac{2\omega_c}{\omega_{\rm BH}}} \simeq \frac{2\alpha_s\,N_c}{\pi}\cdot\frac{L}{\lambda_R} \sim \frac{L}{\lambda_R}\,,$$

where we have used $2\omega_c/\omega_{\rm BH} \simeq (L/\lambda)^2$ and introduced the mean free path for the parton R (quark in our case) which is related with the gluon mean free path by $C_R\lambda_R = N_c\lambda$.

5. Illustrations

5.1 Shift function and subleading (hard gluon) effects

If the integrated gluon multiplicity in (3.4) depended on a single variable ω/ω_c, the quenching factor would have been a function of a dimensionless ratio $X = \frac{p_\perp}{n\omega_c}$. In reality, such a scaling holds only approximately. Indeed, the gluon radiation spectrum (2.6) and, therefore, the multiplicity N, depend on two energy ratios, ω/ω_c and $x = \omega/p_\perp$. The latter takes care, in particular, of the phase space restriction[5] on the energy of radiated gluons, $dI/d\omega \propto N \propto \Theta(1-x)$.

According to (4.1), the main contribution to quenching originates from the region $x \sim \omega_1/p_\perp \sim 1/n \ll 1$. The correction coming from "hard" gluons with $x \lesssim 1$ turns out, however, to be relatively large, $\mathcal{O}\left(1/\sqrt{n}\right)$. To take into account non-soft corrections, in the numerical evaluation we supply the spectrum (2.6) with a factor $(1-x)\Theta(1-x)$ as given in [12, 13].

[5]here we use an approximate equality of $p_\perp$ to the total jet energy

JHEP09(2001)033

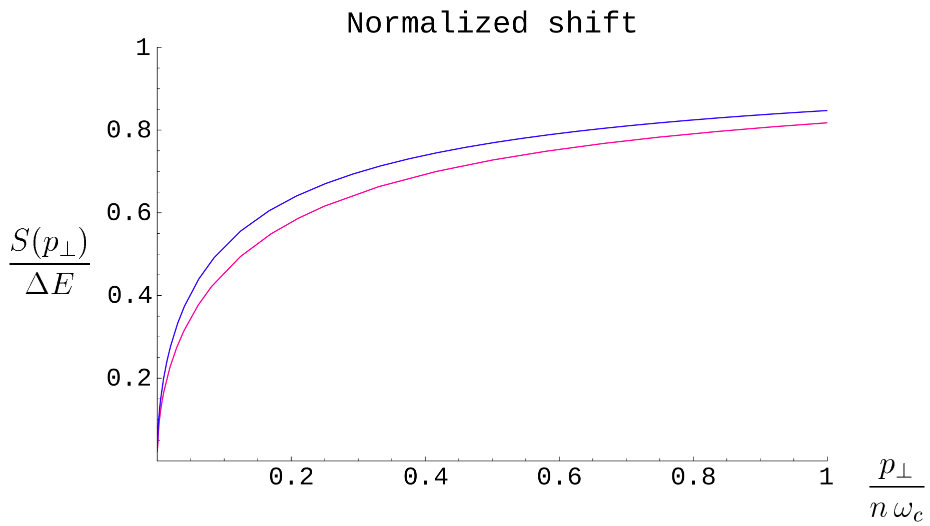

Figure 1: Shift $S(p_\perp)$ as a function of $X \equiv \frac{p_\perp}{n\omega_c}$ for $n = 4$ (lower curve) and $n = 10$ (upper curve).

The effect of the hard gluon correction is illustrated in figure 1 where we plot the shift normalized, for convenience, by $\Delta E = \frac{\pi}{4}\,\alpha\omega_c$ (the BDMPS mean energy loss [11]).

These subleading corrections can be treated analytically. With account of the phase space restriction, $\omega < p_\perp$, and the "hard" factor $(1-x)$ in (2.6) the integrated multiplicity (2.10) gets modified as

$$N(\omega) \simeq \alpha \left\{ \sqrt{\frac{2\omega_c}{p_\perp}} \left[\frac{1}{\sqrt{x}} - 2 + \sqrt{x} \right] + \ln 2 \left[\ln x + 1 - x \right] \right\}, \qquad x = \frac{\omega}{p_\perp}. \tag{5.1}$$

$(N(\omega \geq p_\perp) = 0)$.

The energy integral for the shift function (3.5b) then becomes

$$\begin{aligned} S(p_\perp) &= p_\perp \int_0^1 dx\, N\left(x\, p_\perp\right) e^{-n\,x} \qquad (5.2) \\ &\approx \alpha\omega_c \cdot \left\{ \sqrt{2\pi \frac{p_\perp}{n\,\omega_c}} \left(1 - \frac{2}{\sqrt{\pi\, n}} + \frac{1}{2n} \right) - \frac{p_\perp}{n\,\omega_c} \ln 2 \left[\ln n + \gamma_E - 1 + \frac{1}{n} \right] \right\}, \end{aligned}$$

where we have omitted exponentially small terms $\mathcal{O}\left(\exp(-n)\right)$. We see that the hard correction is rather large, $\mathcal{O}\left(1/\sqrt{n}\right)$, and significantly modifies the behaviour of the shift even in the strong quenching limit, $X = \frac{p_\perp}{n\omega_c} \ll 1$.

The exact numerical evaluation of $S(p_\perp)$ is compared with the approximate formula (5.2) in figure 2 for $n = 4$. The comparison shows that even for the smallest n value the approximation based on $p_\perp \ll \omega_c$ turns out to be rather good up to $p_\perp = 2 \div 2.5\omega_c \sim \sqrt{n}\,\omega_c$.

JHEP09(2001)033

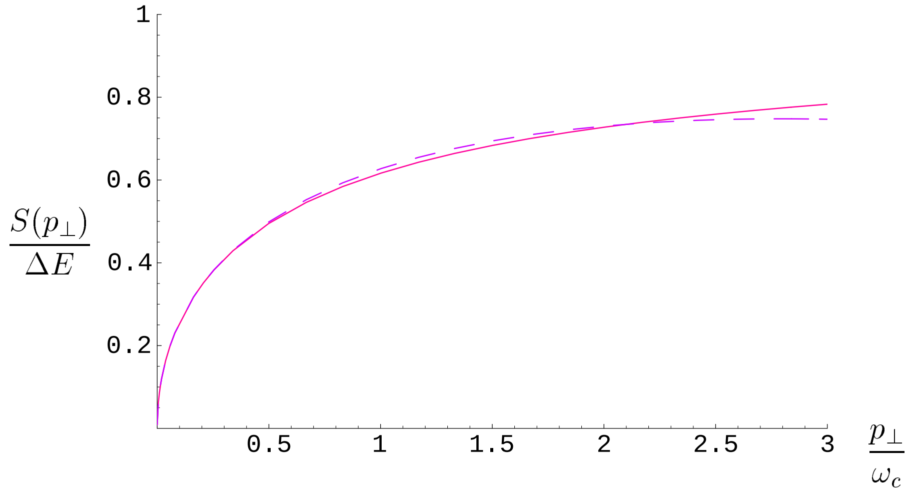

Figure 2: Shift function for $n = 4$ (solid) and its analytic approximation (5.2) (dashed).

5.2 Quenching factors and infrared sensitivity

The estimate (4.6) shows that in the presently available RHIC range $p_\perp < 6\,\text{GeV}$ a reliable quantitative prediction of quenching can be hardly made. It is the soft "singularity" of the LPM spectrum $\omega dI(\omega)/d\omega \propto 1/\sqrt{\omega}$ that causes instability of the perturbative QCD description.

At the same time, this very same instability makes the study of quenching in the $p_\perp \sim 20\,\text{GeV}$ range, which is accessible at RHIC, the more interesting and valuable. In this region the characteristic gluon energies are comfortably large, $\omega_1 \sim 2\,\text{GeV}$, while the quenching factor at the same time is still quite sensitive to much smaller energies deep into the infrared domain.

To illustrate this point in figure 3 we show the expected quenching factors for the hot medium ($\hat{q} = 0.2\,\text{GeV}^3$, sizes $L = 2$ and $L = 5$ fm) as a function of an infrared gluon energy cutoff. A sharp cutoff is not realistic and can be used only as a means of quantifying sensitivity of the answer to the low momentum region. In reality, what matters for quenching is the spectral properties of the (hot) medium which determine how the gluons with energies of the order of ω_{BH} are being produced.

The label "n floating" in figure 3 (and below) means that for this $p_\perp$ range we have used the realistic fit to the vacuum spectrum provided by the PHENIX collaboration [3],

$$\frac{d\sigma^{\text{vacuum}}(p_\perp)}{dp_\perp^2} = \text{const} \cdot (1.71 + p_\perp\,[\text{GeV}])^{-12.44} ,$$

and evaluated the effective exponent $n(p_\perp)$ according to (1.1).

In figure 4 the quenching factors are shown for large transverse momenta, where we have set $n = 4$, the asymptotic value. A mismatch between the Q values at the common point $p_\perp = 20\,\text{GeV}$ in figures 3 and 4 is due to the much stronger bias effect in the former case: $n \approx 12 \gg 4$. Knowing the $p_\perp$ dependence of the vacuum

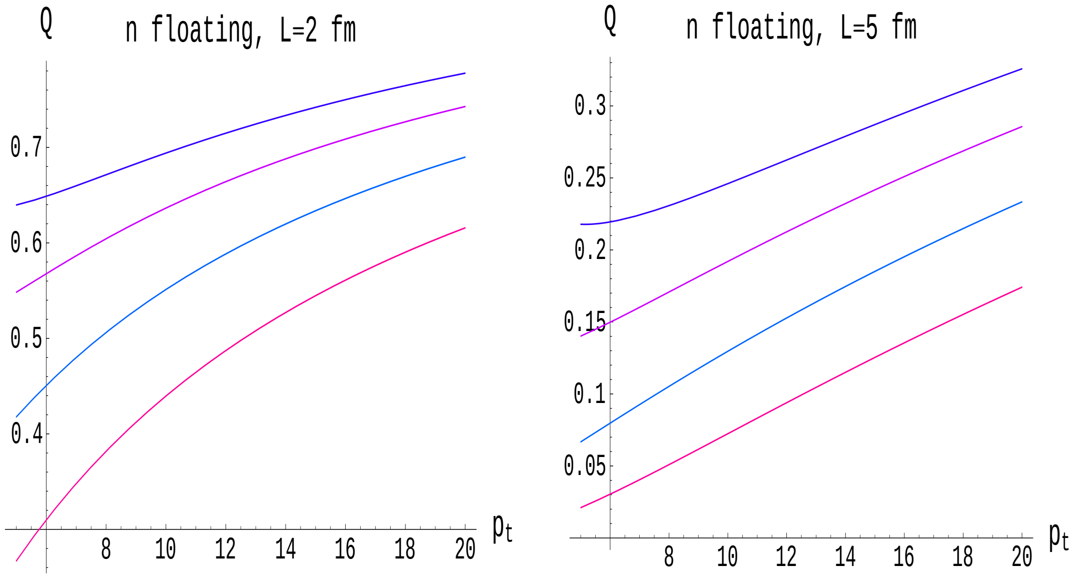

Figure 3: "Infrared" dependence of the quenching factor for hot medium. The curves (from bottom to top) correspond to the gluon energy cuts 0, 100, 300 and 500 MeV.

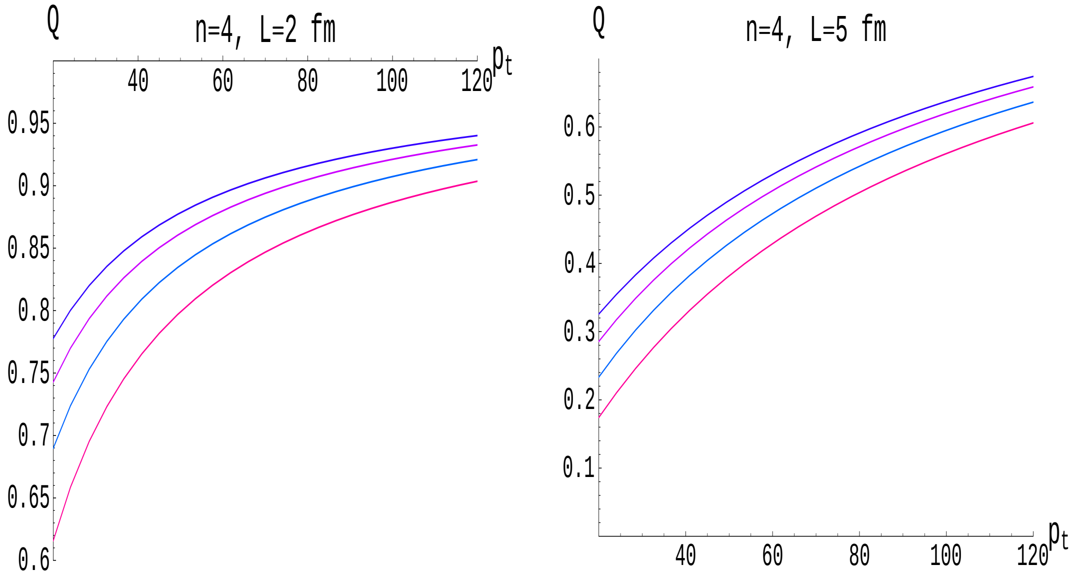

Figure 4: Quenching factors for hot medium. (The curves are the same as in figure 3.)

production cross section is, obviously, a necessary ingredient of a reliable quenching prediction.

Comparing the two figures we remark that at large transverse momenta (and smaller n) the sensitivity to the infrared physics gets naturally reduced.

In figure 5 for the sake of comparison the magnitude of quenching for a final state matter with the transport coefficient equal that of the cold nuclear matter, $\hat{q} = \hat{q}_{\rm cold} = 0.01\,\mathrm{GeV}^3$, is displayed.

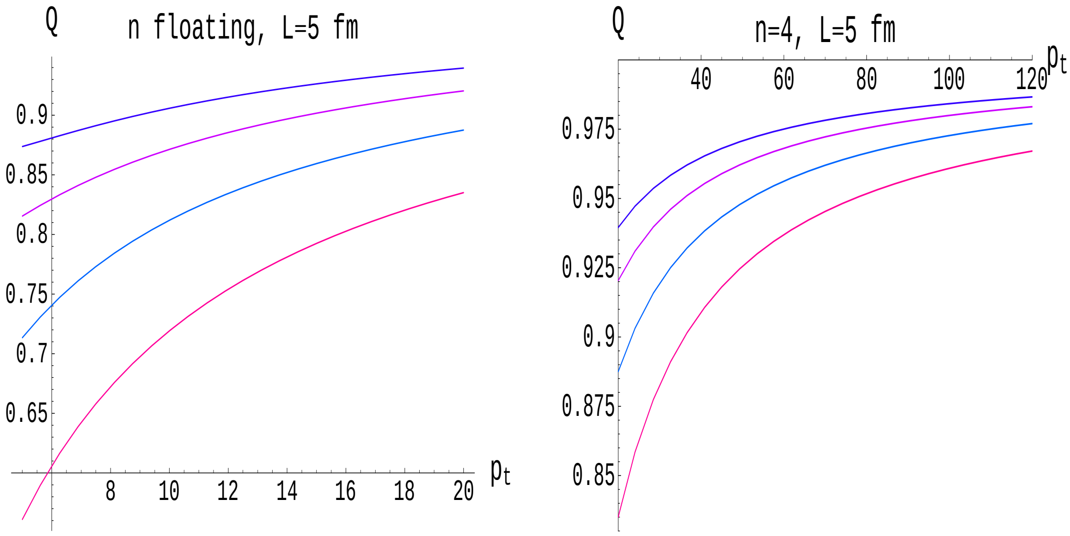

Figure 5: Quenching factors for cold medium. (The curves are the same as in figure 3.)

6. Conclusions

In this paper we developed the perturbative QCD description of the suppression (quenching) of inclusive production of hadrons with large transverse momenta $p_\perp$ in a medium. We did not address the Cronin effect that *enhances* particle yield at moderately large $p_\perp$. A simple analysis shows that parton rescattering effects both in the initial and in the final state (transverse momentum broadening) should die out as $\mathcal{O}\left(\hat{q}L/p_\perp^2\right)$, to be compared with the $\mathcal{O}\left(L\sqrt{\hat{q}/p_\perp}\right)$ behaviour characteristic for quenching.

We found that quenching is related with the multiplicity of medium induced (primary) gluons with energies larger than the characteristic energy ω_1 as

$$-\ln Q(p_\perp) = \int_0^\infty dz\, e^{-z}\, N(\omega_1 z)\,,$$
$$\omega_1(p_\perp) \equiv -\left[\frac{d}{dp_\perp}\ln\frac{d\sigma^{\mathrm{vacuum}}}{dp_\perp^2}\right]^{-1} \simeq \frac{p_\perp}{n}\,,\; n \gg 1\,. \tag{6.1}$$

One of the results of this study is the observation that the "shift" parameter S in the commonly used parametrization (3.6),

$$\frac{d\sigma^{\mathrm{medium}}(p_\perp)}{dp_\perp^2} \simeq \frac{d\sigma^{\mathrm{vacuum}}(p_\perp + S)}{dp_\perp^2}\,,$$

equals neither the *mean* medium induced energy loss, $S = \Delta E \propto L^2$, nor $S = \mathrm{const}\cdot L$. In fact, all over a broad kinematical region where quenching is sizeable, $-\ln Q(p_\perp) = \mathcal{O}\left(1\right)$, the shift increases with $p_\perp$ as

$$S(p_\perp) \simeq \frac{2\alpha_s C_F}{\pi}\sqrt{2\pi\,\omega_c\,\omega_1} \propto L\sqrt{p_\perp}$$

and equals *twice* a typical energy the quark looses to induced radiation.

We also observed that the particle (jet) quenching phenomenon is sensitive to the character of the *distribution* in the energy loss $D(\epsilon)$ (in particular, to the energy region $\bar{\epsilon} \sim \alpha^2\omega_c \ll \omega_c$, where $D(\epsilon)$ has a maximum, see (2.17)), rather than to its high-energy tail $\epsilon \sim \omega_c$ which determines the mean energy loss ΔE. The latter is dominated by a rare ($\mathcal{O}(\alpha_s)$) single gluon emission with maximal available energy, $\epsilon \approx \omega \sim \omega_c$.

Though formally a collinear- and infrared-safe quantity, we found the quenching factor $Q(p_\perp)$ to be highly sensitive to the region of small gluon momenta, especially for $p_\perp \lesssim 20\,\mathrm{GeV}$. The two effects that contribute to this are the bias ($n \gg 1$) and the LPM suppression which makes the gluon energy spectrum singular at $\omega \to 0$. On the one hand, the region where the characteristic gluon energy ω_1 becomes comparable with $\omega_{\mathrm{BH}} \sim 300\,\mathrm{MeV}$ (which regulates the transition from the Landau-Pomeranchuk-Migdal to the Bethe-Heitler regime) the pure perturbative treatment is hardly applicable. On the other hand, the experimental (and possibly theoretical) studies of the momentum region corresponding to $\omega_1 \sim (1 \div 2)\,\mathrm{GeV}$ which corresponds to $p_\perp = 10 \div 20\,\mathrm{GeV}$ (for $n \simeq 10$) where the Cronin effect is expected to have disappeared, will provide an important information about the spectral properties of the final state medium produced in heavy ion collisions.

The final remark concerns our treatment of a medium as uniform and static, which is obviously not a realistic approximation. For "hot" medium characterized by a large transport coefficient $\hat{q} \sim 0.2\,\mathrm{GeV}^3 \simeq 5\,\mathrm{GeV\ fm}^{-2}$ we estimate the characteristic lifetime of gluons with energies $\omega_1 \sim p_\perp/n$ that determine quenching as

$$t_1 \sim \sqrt{\frac{p_\perp}{n\,\hat{q}}} \quad \sim \left(\sqrt{\frac{p_\perp}{5 \cdot n\,\mathrm{GeV}}}\right) [\mathrm{fm}]\,.$$

In the most interesting and practically important situation this lifetime is small compared with the size of the medium, $t_1 \ll L$. In these circumstances it looks plausible that one will be able to calculate the quenching factor by employing the basic equations (2.13) with the dynamical *position-dependent* induced gluon distribution $dI(\omega, z)/d\omega dz$ given in [28].

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